Exercise Sheet 5

Exercise 1. Let G be a finite group and let H be a subgroup of G. Let $A := \mathrm{Coind}_G^H(\mathbb{Z})$ where H acts trivially on $\mathbb{Z}$. Is $H^1(G, A)$ always 0? What about $H^2(G, A)$? (If so, prove it. If not, give a counter-example.)

Exercise 2. Let $G = \mathrm{PSL}(2, \mathbb{Z})$, and let A be a G-module. Show that for every $i \geqslant 2$ and for every $x \in H^i(G, A)$, we have $6x = 0$. (Hint: note that G contains a free subgroup of index 6.)

Exercise 3. Prove the following proposition whose proof we omitted in class:

Let $q \in \mathbb{Z}_{>0}$. Suppose $H^i(H, A) = 0$ for any $1 \leqslant i \leqslant q-1$, then the sequence

$$0 \to H^q(G/H, A^H) \xrightarrow{\mathrm{Inf}} H^q(G, A) \xrightarrow{\mathrm{Res}} H^q(H, A)$$

is exact and

$$\mathrm{Inf}\colon H^i(G/H, A^H) \xrightarrow{\sim} H^i(G, A)$$

for any $0 \leqslant i \leqslant q-1$.