**Exercise Sheet 1**

**Exercise 1.** Let $K$ be a field and let $|\cdot|$ be an absolute value on $K$. Give an example where $|K^*|$ is discrete in $\mathbb{R}_{>0}$ but $|K|$ is not discrete in $\mathbb{R}$.

**Exercise 2.** Let $K$ be a field and let $|\cdot|_1, ..., |\cdot|_n$ be non-trivial inequivalent absolute values on $K$. Let $r_1, ..., r_n$ be real numbers. Suppose

$$|a|_1^{r_1} \cdots |a|_n^{r_n} = 1$$

for all $a \in K^*$. Prove that $r_i = 0$ for any $i$. (In other words, there is NO finite product formula.)

**Exercise 3.** Let $K$ be a field and let $|\cdot|_1, ..., |\cdot|_n$ be non-trivial inequivalent absolute values on $K$. Denote by $K_i$ the completion of $K$ with respect to $|\cdot|_i$. Prove that the image of $K$ under the diagonal embedding $K \hookrightarrow \prod K_i$ is dense, i.e. for any $n$-tuple of elements $a_i \in K_i$ $(i = 1, ..., n)$ and any $\epsilon > 0$, there exists an element $a \in K$ s.t. $|a - a_i|_i < \epsilon$.